

**mechanics
monograph
M-5**

Mechanics Division
of the
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for Engineering Education

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Demonstration of Consequences of the Continuum Hypothesis

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The significance of several crucial assumptions within the theory of deformable bodies can be demonstrated via simple observations of motion of a crack in a bar of a structured material. The demonstration is based on a null experiment of a sort requiring no instrumentation. The demonstration is therefore ideally suited for the classroom environment.

In developing concepts of stress and strain within an elementary course in the mechanics of deformable bodies, one relies heavily on arguments based on free-body diagrams of differential elements of material. Required assumptions, including those of continuity of the medium and the nature of the interactions across the boundary of the element, (i) are often not introduced at all, or (ii) are present implicitly only in the limiting process, or (iii) are soon forgotten. These assumptions are important not only in terms of a rational development of elementary mechanics, but also in view of modern uses of composite materials.

Consequences of the continuum hypothesis and its failure may be illustrated by means of a bar of flexible material, of square or rectangular cross section. Such a bar, if sufficiently large, is also useful for demonstrations of principal and anticlastic curvatures in bending, and warp of the cross sections in torsion. For the purpose of this article, consider two such bars, identical in size and shape. Let one be made of rubber and the other of polymeric foam such as that used as packing material.

Consider a differential element at the corner of the bar, as shown in figure 1. The surface traction upon the lateral surface is zero for torsional loading of the bar. By virtue of the symmetry of the stress tensor, the complementary shear stress must be zero, as shown in figure 1. Since all components of the shear stress vanish at the corner, the shear strain must also vanish. By such arguments one can show that the cross sections of a rectangular bar in torsion must undergo warp: see, e.g. F.P. Beer and E.R. Johnston, Jr., *Mechanics of Materials*, McGraw Hill, New York, 1981. Consider, however, a similar microelement in a material with a lattice type structure, as shown in figure 2. The

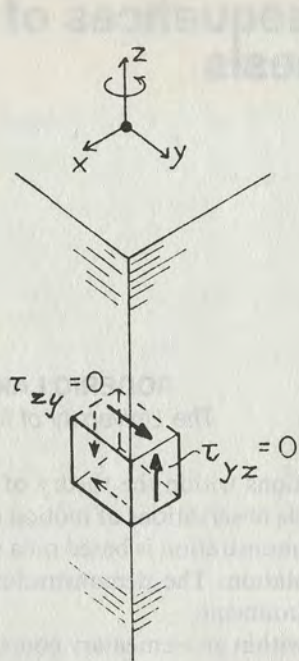


Fig. 1. Free-body diagram of corner element of unit width in an elastic continuum. Only those tractions involved in the rotational equilibrium of the element about the x axis are shown.

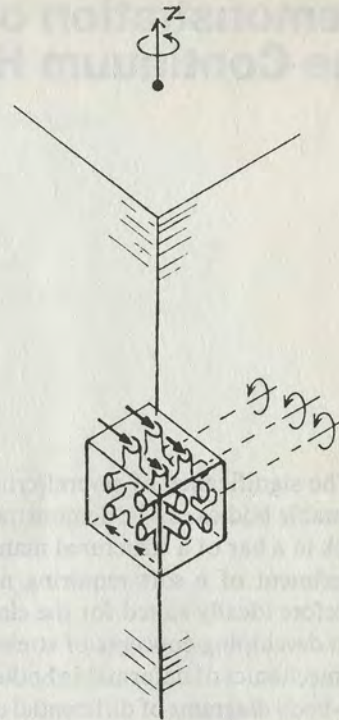


Fig. 2. Free-body diagram of corner element in a structure.

struts in the lattice can support a moment as well as a force, as shown. The moment must be nonzero since the transverse struts undergo a twist. For the micro-element to be in equilibrium, a nonzero shear force must be transmitted through the longitudinal struts as shown.

The corner elements of an elastic material and a lattice structure, then are predicted to behave quite differently. To illustrate this difference, make a small nick in the corner of the rubber bar and a similar nick in the corner of the foam bar. When the rubber bar is twisted, the crack is not expected to open in mode III (in which the faces of the crack shear parallel to each other and parallel to the crack front) since there is no stress in the region of the crack. While this is strictly true only for an infinitesimal crack, the crack opening is negligibly small if the crack is sufficiently short compared with the bar width, as shown in figure 3. The situation is different in the foam bar. The corner crack opens noticeably in mode III when the bar is twisted, as shown in figure 4. Here the corner of the bar has been made more visible by gluing black thread to it with

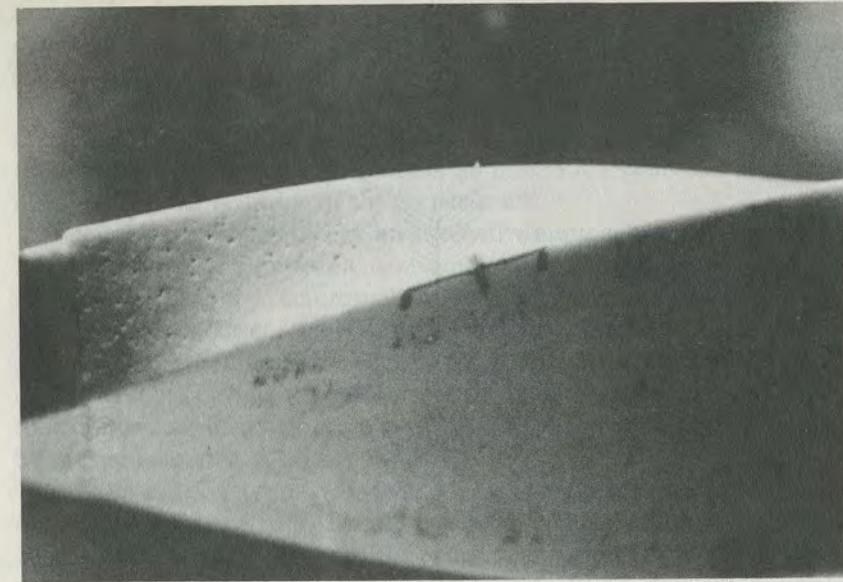


Fig. 3. Behavior of a crack in the corner of a bar of rubber in torsion.

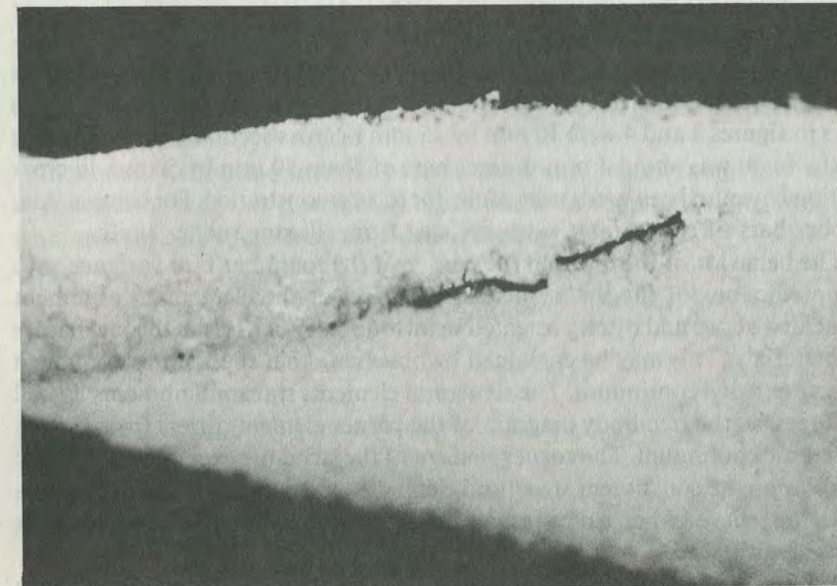


Fig. 4. Behavior of a crack in the corner of a bar of foam in torsion.

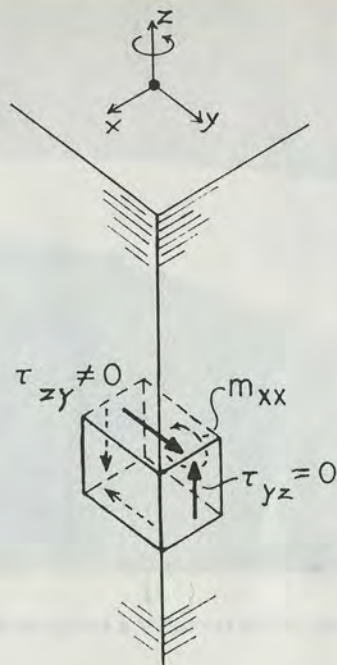


Fig. 5. Free-body diagram of a corner element of a bar in torsion, made of a continuum which supports couple stresses.

rubber cement. A line can also be drawn with ink as was done with the rubber bar, but the line tends to be rather broad, due to the structure of the foam. The bars in figures 3 and 4 were 10 mm by 25 mm in cross section and the cell size of the foam was about 1 mm. Larger bars of foam 50 mm by 50 mm in cross section have also been used successfully for this demonstration. For comparison, rubber bars of comparable size were cast from silicone rubber caulk.

The behavior of the crack in the corner of the foam bar is at variance with the predictions of the theory of elasticity and of the elementary argument presented above and often presented in introductory courses in the mechanics of materials. This may be explained by observing that the foam is actually a structure, not a continuum. The structural elements transmit moments as well as forces, so the free-body diagram of the corner element differs from that of the elastic continuum. The corner element of the structure experiences nonzero loads upon the constituent structural elements, when the bar is twisted. These loads are relieved when a nick is made in the corner, so the surfaces of the nick become displaced.

In introductory courses in the mechanics of deformable bodies, the above demonstration has been used following the analysis of torsion and the discussion of warp of cross sections in members of noncircular cross section. The students are first challenged to explain the corner crack phenomenon, then an

explanation is given, followed by a discussion of composite materials. The demonstration is found to not only be useful pedagogically, but also to excite considerable interest in composites.

The demonstration of the corner crack shear phenomenon has also been used in graduate courses in the theory of elasticity and in oral examinations for the Ph.D. Advanced graduate students are no more successful than sophomores in explaining the behavior of the corner crack.

In courses in elasticity theory, an alternative approach is possible. The free-body diagram of the corner element can be redrawn with the set of moments upon the struts in the structure replaced by a distributed moment per unit area, or couple stress. The ensemble of forces upon the struts is replaced by a force per unit area or the conventional stress. Recall that the differential element was chosen to be a cube of unit side. The corner crack demonstration can then serve to introduce the idea of couple stresses and Cosserat continua in advanced courses in elasticity.

If the idea of couple stress is admitted, the phenomenon of shear of the corner crack can be treated within the context of continuum theory. The matter is interpreted not as a failure of the continuum hypothesis but as a failure of the hypothesis that the interaction across the surface is describable solely in terms of a force vector: a couple vector is also required.

From this point of view, a nonzero state of stress and couple stress is permitted to occur in the corner element. These stresses are then relieved locally when a nick is made, so the faces of the nick undergo relative displacement.

Some writers (e.g. A.P. Boresi and P.O. Lynn, *Elasticity in Engineering Mechanics*, Prentice Hall, NJ, 1974) prefer to introduce the topic of couple stresses at the senior or first year graduate level, however we have not attempted to do this. For those who favor such an approach, the demonstration of corner crack shear can be of use in developing a physical perception of the idea of couple stress.

In summary, the demonstration of shear at a corner crack is found to be useful in stimulating students to critically examine the assumptions underlying the development of the mechanics of deformable bodies.

Lo-Tech Widgets for Vibrations Classes

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INTRODUCTION

When answering the course evaluation question: "What could the instructor do to improve this course?" A frequent response was, "More toys!" Perhaps your students are more mature than mine, but I doubt it. They all like to play with gadgets that confirm their nascent faith in some arid theory. And we all can recall experiments, in which we "got our hands dirty," long after the theory and problem sets melted into a muddled mass in some back drawer of our minds.

I had three incentives motivating me to develop low-tech toys for my senior vibrations course. First, I had neither taken vibs as a student, nor had I set foot in a vibrations lab. However, learning a course by teaching it is probably as familiar an experience for most of you as it is for me. And it does have its advantages. In this case it gave me the incentive to test out the theory. And I was pleasantly surprised to find that elaborate apperati were unnecessary.

This brings me to the second motivation. I hate labs! Expensive equipment (only safely handled by a factory technician) turns me off. And reports (ink-printed in their stifling styleless style with the usual inane questions with single answers that generations of students have forced out of the data or argued that they would have gotten had the apparatus worked properly) are pure drudgery. So my discovery of simple tests, using only sticks and string initially, that agreed with and abetted the theory, pleased me enormously.

The third reason was economic. Small private Catholic colleges, like Gonzaga University, are chronically short of cash. Even had I wanted money for the high-tech equipment of a "real" lab, the probability of it dropping in my lap would only have been slightly greater than that of the Second Coming.

SOME BACKGROUND INFORMATION

My non-lab typically follows the following format. First, we proceed through the theory, with free body diagrams and differential equations concluding with the natural and unnatural frequencies of vibration. Next, we add in the physical

constants and calculate the numerical theoretical results. Finally, using only stop watches, (a feature of many digital watches today) several student volunteers/hecklers gather the data. (Usually this is based on ten cycles of the device and averaged.) So, as you can see, the level of sophistication is nearly that of the 18th century kite flyer.

This imposes one severe constraint on most of these gizmo's, viz. their rate of oscillation must be slow enough to be countable by your average student.

Relationships between reality and theory are a critical and often missing link in many T&AM courses. One cannot directly see things such as: heat flowing, entropy increasing, stress relaxing, or even rapid vibrations. Instruments assure us that these things exist. And clearly such instruments are important and essential aids to us. But at the same time they are also barriers. Another level of complexity and separation has been inserted between the embryonic real-world engineer and the subject of his attentions. Thus, I only resort to the use of sophisticated equipment when all other attempts to "tell and then show" have failed.

CENTER OF PERCUSSION

The course begins with a review of engineering dynamics. One interesting discovery therein is labeled the "center of percussion." It is an easily demonstrable phenomenon. As you will recall, a blow at the center of percussion produces no horizontal force at the support (see Fig. 1a). Thus if the rigid support is replaced by a string as shown in Fig. 1b, the blow to the center of percussion will only cause rotation about the point where the string is attached, i.e., no horizontal motion. After calculating the distance, y , to this point we are ready to perform the "experiment" (ignoring relativistic effects). Sure enough, it works. We find the point where the top of the stick stays put. And on measuring, we find that it is indeed very close to what the theory predicted.

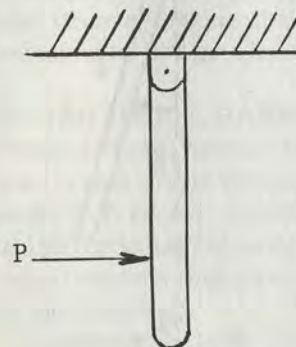


Figure 1a

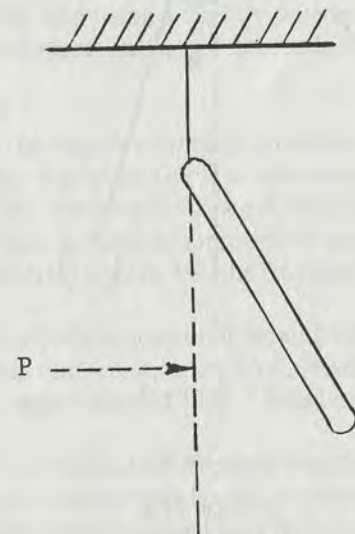


Figure 1b

SLOW SIMPLE HARMONIC MOTION

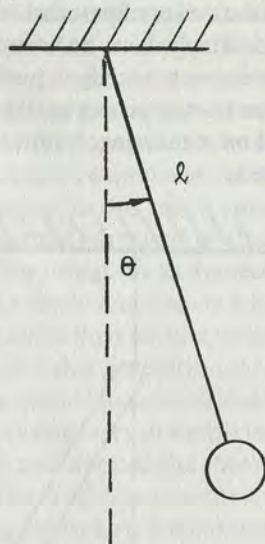
I'm certain that the uses for a pendulum are myriad. The one that I suggest requires that the student picture himself on a peaceful Pacific atoll, lying in a hammock stretched between two palm trees, waiting for his 3-minute eggs to cook. The timing device is, of course, a pendulum of his own design. The simple pendulum, the compound pendulum, and the torsion pendulum are all considered.

The theory, assuming small angles, point masses, and no friction yields equations of motion that are all of the same form, differing only in the coefficient of the dependent variable. The equations and the solutions for the natural frequencies, ω_n , and the periods, τ , are given in the Figures 2, 3, and 4.

The writing of the differential equations is an essential part of any vibrations course. But the process should not stop there. Calculating co-efficients and checking units and definitions is critical.

The students are asked, in class, to calculate the length of the string such that the period of the simple pendulum will be one second. Also, they determine the period for the yardstick compound pendulum. Finally, they find the period for the torsion pendulum—which requires some head scratching to recall what J , G , and I_o are, and then calculating them for the physical setup.

At last the moment arrives and the tests are run. All agree, surprisingly well (to me at least), with the theory. A solid connection to the real world has been made.

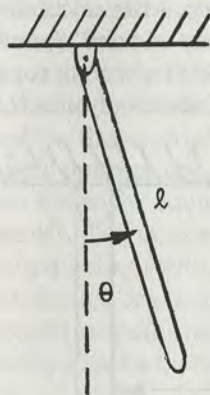


$$\ddot{\theta} + \frac{g}{l} \theta = 0$$

$$\omega_n = \sqrt{g/l}$$

$$\tau = 2\pi \sqrt{l/g}$$

Figure 2

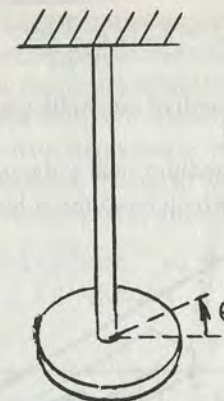


$$\ddot{\theta} + \frac{3g}{2l} \theta = 0$$

$$\omega_n = \sqrt{\frac{3g}{2l}}$$

$$\tau = 2\pi \sqrt{\frac{2l}{3g}}$$

Figure 3



$$\ddot{\theta} + \frac{JG}{LI_o} \theta = 0$$

$$\omega_n = \sqrt{\frac{JG}{LI_o}}$$

$$\tau = 2\pi \sqrt{\frac{LI_o}{JG}}$$

Figure 4

To improve on this connection, I assign one homework-lab problem. Their task is to write the differential equation and calculate the period for a partially filled can or jar bobbing in water. After they have these expected results, they then must perform the physical test to confirm, or dispute, their work. There is a certain amount of grumbling, but there is an even greater amount of satisfaction. (Even eyeballing dimensions with a ruler and using a slightly tapering peanut butter jar I got a less than 5% error—and a great amount of satisfaction!)

FASTER SIMPLE HARMONIC MOTION

The limitation of having cycles slow enough to be visually countable, constrains your lo-tech widget severely. One way to relax this constraint is to use sound as the basis of your measuring instrument. If a device will vibrate in the 100 to 1500 cycle/second range, you can easily hear it. And by comparison with a choir master's pitch pipe one can determine, to within 2 or 3%, the frequency of the vibration.

It seems strange to me that most engineering vibrations texts omit the auditory area (leaving it to a separate course in acoustics that few students take). Nevertheless, in an attempt to extend my widget's ranges beyond 3 Hz. I asked our shop man to make me a music box.

The cantilevered saw blade, (see Figure 5), is mounted on an empty wooden box with a hole cut in one end. By varying the cantilevered length, a variety of notes can be produced. The natural frequency is determined as in the other cases, and is:

$$f_n = 3.515 \sqrt{\frac{EI_s}{\mu \ell^4}}$$

where μ is the weight per unit length of the cantilever. The classroom test is similar to those previously described.

(Note: If you wish to get into medium-tech widgets, it would be a simple matter to use a strobe on any device which has a fairly large amplitude of vibration.)

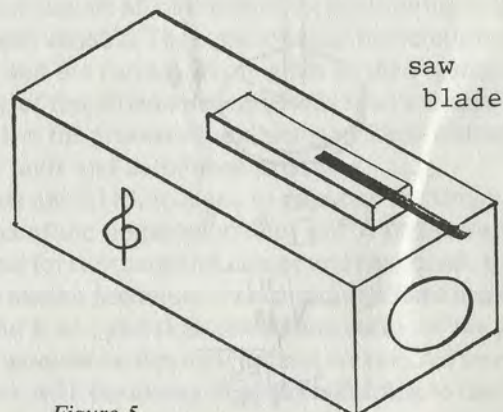


Figure 5

MODES OF VIBRATION

Another use for your basic string and stick toy is to demonstrate modes of vibration. With a yardstick and half a yard of string, the resultant modes of vibration have periods of 2.0 and 0.53 seconds (both easily countable). Figure 6 shows the extreme positions as the top and bottom of the stick oscillate.

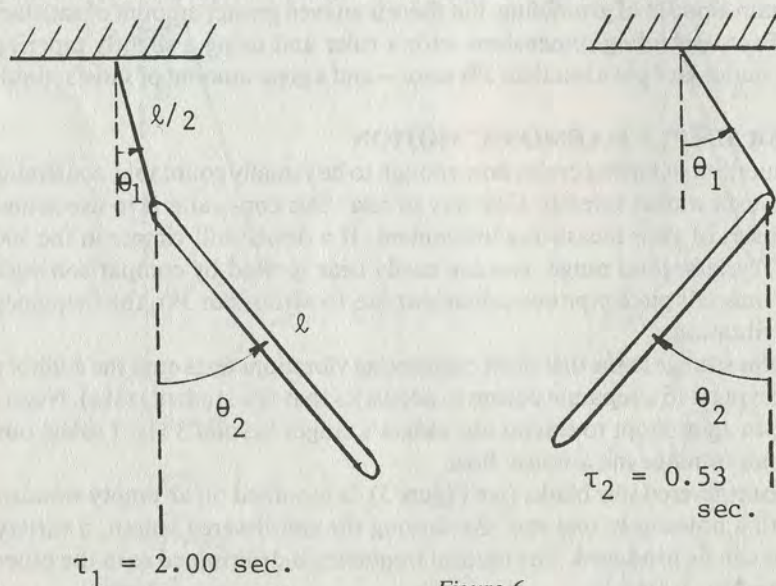


Figure 6

ACCELEROMETER/VIBROMETER

My final lo-tech suggestion is an attempt to approximate commercially available instruments. These measure either accelerations of the base beneath the instrument or the magnitude of low frequency vibrations such as earthquakes. Both are based on the same physical setup viz. a device with natural frequency, ω_n , resting on a vibrating surface with frequency ω . In Figure 7, the cantilever with mass, m , attached (at some variable distance) will have the natural frequency ω_n . The supporting surface vibrates with amplitude, b , and frequency, ω .

"Metronome" with natural frequency

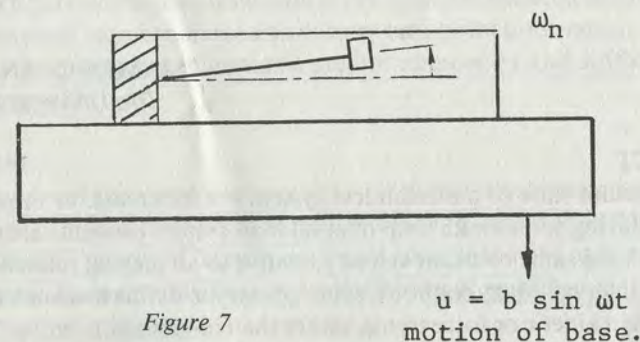


Figure 7

Again the music department may come to our aid. The device appears very similar to a metronome laid on its side. When the ratio of frequencies, $\omega/\omega_n > 10$, the relative motion between the mass and the frame should be nearly the same as b , the magnitude of the vibration of the support. And when $\omega/\omega_n < 0.1$, the product of ω_n^2 and the relative motion should approximate the acceleration of the support.

I have not, as yet tried to make this device, but I don't see any serious difficulties in doing so. The problem in using it will more likely arise in bringing the vibrating support base into the classroom for the test. (If any of you do try this, I would be pleased to learn of your success or frustrations.)

CONCLUSION

I am only bounded in my pursuit for more and better lo-tech widgets by the limits of my creativity and by the available time. But it's worth it. These toys, i.e., things to play with, enjoy and learn from, are probably more fun for me than for my students. But my excitement is transmitted, and it adds an essential dimension to their understanding and retention of the material. In short, I give off good vibes.

On the Concept of Mechanical System Equilibrium

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ABSTRACT

The equilibrium state of a mechanical system is a kinematic or motion state that exists during some finite time-interval if all system elements are at rest or moving with the same constant velocity relative to an inertial reference frame during that interval. Many textbook writers, however, define mechanical system equilibrium in kinetic or force terms, rather than in kinematic terms. The purposes of this paper are to emphasize (1) that these two definitions are in fact consistent and interchangeable only when the mechanical system of interest is a single particle moving at non-relativistic speeds, and (2) that, in general, the kinetic definition of equilibrium is misleading and substantially incorrect. Two simple examples are cited to illustrate the contradictions that can arise when the kinetic definition is used. Finally, an attempt is made to explain why this misconception of equilibrium persists, and suggestions are offered to help minimize its adverse effect where its presence cannot be easily eliminated.

KEY WORDS

Equilibrium; necessary conditions; sufficient conditions; equipollence.

INTRODUCTION

The concept of mechanical system equilibrium rests at the very foundation of the science of mechanics. The importance to engineers of having a thorough understanding of this concept is reflected by the fact that most accredited engineering science and engineering technology programs include a statics course as one of the basic requirements for a bachelor's degree.

Equilibrium, as a general concept, is familiar to most of us, and although its meaning is sharpened in the mechanics context, its general intuitive connotations are preserved. In simple terms, a mechanical system is in equilibrium if all system elements are at rest or moving with the same constant velocity in the same inertial reference frame. Thus, equilibrium is properly defined as a kinematic or motion state of a mechanical system, a state where disturbing or

disruptive influences are either not present, or are effectively nullified by counter-acting influences. Specifically, a mechanical system is in an equilibrium state during a finite time-interval $\Delta t > 0$ if all constituent particles (or differential elements) of that system move with the same constant velocity relative to the same inertial reference frame during that interval. In the preferred inertial reference frame, all constituent elements of a mechanical system that are in equilibrium is at rest.

Many mechanics textbook writers, however, define mechanical system equilibrium in kinetic or force terms, rather than in kinematic terms.^{1,2,3,4,5} They usually restrict their attention to particles and rigid bodies, stating that (i) a particle is in equilibrium if the resultant external force acting on it vanishes, and (ii) a rigid body is in equilibrium if the resultant external force and the resultant external moment about an arbitrary base point both vanish. They often add that these kinetic requirements are both necessary and sufficient conditions for equilibrium.

PURPOSE

The purposes of this paper are to emphasize that (1) the kinematic and kinetic definitions of equilibrium are in fact consistent and interchangeable only when the mechanical system of interest is a single particle moving at non-relativistic speeds, and (2), in general, the kinetic definition of equilibrium is misleading and substantially incorrect.

SINGLE PARTICLE EQUILIBRIUM

Restricting the discussion here to motion at non-relativistic speeds, it is not difficult to prove that for a single particle P of constant mass m, the necessary and sufficient kinetic condition for P to be in equilibrium is the vanishing, during $\Delta t > 0$, of the resultant external force $\bar{F}$ acting on P.^{6,7,8,9} To prove that this condition is necessary, one needs to show that the existence of the equilibrium state implies that this condition is necessarily satisfied. Using the kinematic definition of particle equilibrium, the existence of the equilibrium state implies that P is moving with constant velocity $\bar{v}$ relative to an inertial reference frame R. Hence, the acceleration $\bar{a}$ of P in R vanishes during Δt , and Newton's second law ($\bar{F} = m\bar{a}$) therefore implies that $\bar{F}$ must also vanish during this time interval. To prove that this condition is sufficient, one needs to show that the vanishing of $\bar{F}$ during Δt implies that P is in equilibrium. Using Newton's second law, the vanishing of $\bar{F}$ during Δt implies that $\bar{a}$ must also vanish, and therefore $\bar{v}$ must be a constant vector during Δt . Hence, by the kinematic definition, P must be in equilibrium during this time-interval.

These arguments show that the vanishing of $\bar{F}$ during $\Delta t > 0$ is both a necessary and a sufficient kinetic condition for P to be in an equilibrium state during Δt . Since it is both a necessary and a sufficient condition for the existence of equilibrium, it may be and often is used as an alternative definition of that system state. However, if it were merely a necessary and not a sufficient condition, or vice versa, it could not be used as an alternative definition of single particle equilibrium.

RIGID BODY EQUILIBRIUM

Many textbook writers define rigid body equilibrium in kinetic terms, stating that a rigid body B is in equilibrium during $\Delta t > 0$ if the resultant external force $\bar{F}$ and the resultant external moment $\bar{M}_O$ about an arbitrary base point O both vanish during that interval.^{1,2,3,4,5} A proof that these two requirements are necessary kinetic conditions for rigid body equilibrium is not difficult to construct.^{6,7,8,9} This proof is of particular interest because it is not based on the rigidity of the system, and it therefore indicates that these two requirements are necessary kinetic conditions for the equilibrium of all mechanical systems. They are therefore of great importance in the analysis of mechanical systems that are known to be in equilibrium. To prove that these conditions are necessary, one simply needs to show that the existence of equilibrium implies that these two requirements are necessarily satisfied. Using the kinematic definition of mechanical system equilibrium, the existence of the equilibrium state implies that all particles P_i of B have the same constant velocity $\bar{v}_i$ in the same inertial reference frame R . Hence, their individual accelerations $\bar{a}_i$ in R all vanish, and Newton's second law then indicates that their individual resultant forces $\bar{F}_i$ must also vanish. Finally, using the definitions of the resultant force $\bar{F}$ and the resultant moment $\bar{M}_O$ about an arbitrary base point O acting on B , it is easy to demonstrate that both $\bar{F}$ and $\bar{M}_O$ must vanish.

It is not difficult to show by example, however, that these two kinetic conditions, namely $\bar{F} = \bar{0}$ and $\bar{M}_O = \bar{0}$, are not sufficient conditions for rigid body equilibrium in particular, or for mechanical system equilibrium in general. To this end, consider first the case when a rigid, homogeneous, right circular

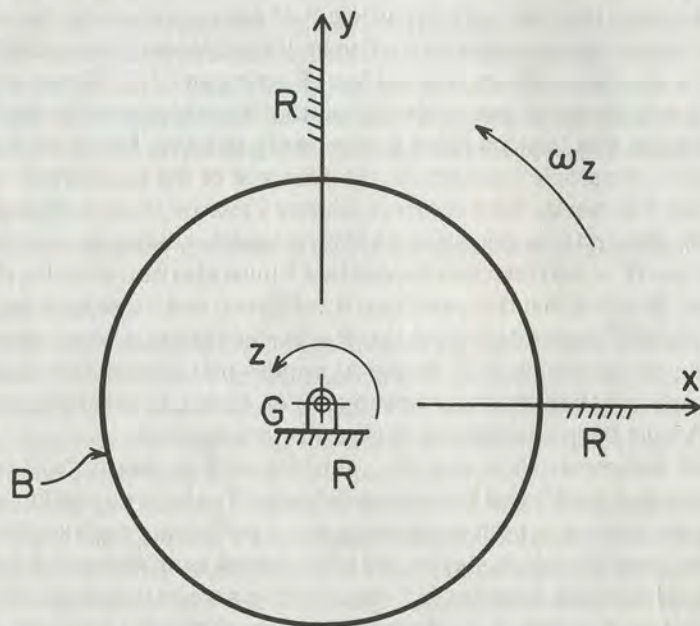


Fig. 1. Rotating, rigid, homogeneous cylinder B , smoothly-pinned at mass center G .

cylinder B of mass m is smoothly pinned in an inertial reference frame R along its z axis of symmetry, and then given an initial angular velocity $\bar{\omega} = \omega_z \bar{k}$ (Figure 1). The mass center G of B lies on the inertially-fixed z axis of B , so the acceleration $\bar{a}_G$ of G in R vanishes. Hence, the principle of linear momentum ($\bar{F} = m\bar{a}_G$) implies that the resultant external force $\bar{F}$ acting on B must also vanish. Furthermore, since B is constrained to rotate about the inertially-fixed z axis passing through G , and since no external forces acting on B exert moments about this axis, the principle of angular momentum ($\bar{M}_G = \bar{H}_G$) implies that the resultant external moment $\bar{M}_G$ about G must vanish, and that ω_z is therefore a constant. Thus, B will continue to rotate forever at its initial angular speed ω_z , and the external force system acting on B will have a resultant force $\bar{F}$ that always vanishes, together with a resultant moment $\bar{M}_G$ about G that also always vanishes. The kinetic definition of rigid body equilibrium is therefore satisfied (i.e., $\bar{F} = \bar{0}$ and $\bar{M}_G = \bar{0}$), yet B is clearly not in a state of rest relative to the inertial reference frame R , or relative to any other inertial reference frame that translates with constant velocity relative to R . In fact, all particles of B at a distance $r > 0$ from the z axis of symmetry are accelerating directly inward toward that axis with an acceleration equal to $r\omega_z^2$. Hence, this example illustrates that $\bar{F} = \bar{0}$ and $\bar{M}_G = \bar{0}$ are simply not sufficient kinetic conditions for the existence of rigid body equilibrium.

As a further example of the inconsistencies and contradictions that can arise when adopting the kinetic definition of equilibrium, consider a non-rigid system consisting of two identical particles, each of mass m , connected by a massless, linearly-elastic spring (Figure 2). If this mechanical system is placed in a resistance-free and gravity-free environment, and if both particles are released from rest in an inertial reference frame R with the spring initially stretched, they will subsequently oscillate forever, with the system mass center G remaining at rest in R at the midpoint of the straight line connecting the two particles at any instant. Is this two-particle system in equilibrium? Irrespective of whether

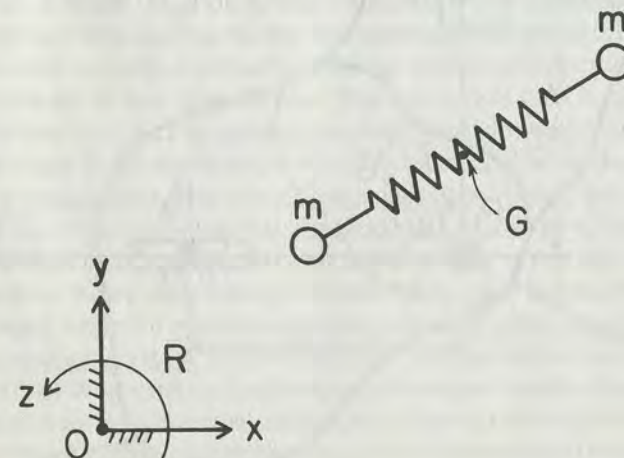


Fig. 2. Vibrating, two-particle system with massless, linearly elastic spring.

the massless spring is included in the system, the sum of the external forces acting on the system always vanishes, and all external forces always pass through the system mass center G , indicating that both $\bar{\mathbf{F}} = \bar{\mathbf{0}}$ and $\bar{\mathbf{M}}_G = \bar{\mathbf{0}}$ at all times. The two kinetic conditions for equilibrium of this mechanical system are therefore satisfied, yet each particle is generally accelerating in R , and is therefore clearly not in an equilibrium state. Since all constituent particles of a mechanical system must be at rest in the same preferred inertial reference frame if that system is to be in an equilibrium state, this system is clearly not in equilibrium. This example again illustrates the fact that these two kinetic requirements are not sufficient conditions for equilibrium.

Since $\bar{\mathbf{F}} = \bar{\mathbf{0}}$ and $\bar{\mathbf{M}}_G = \bar{\mathbf{0}}$ are not sufficient kinetic conditions for mechanical system equilibrium, it is clear that they should not be used to define that system state. Nevertheless, they are commonly-used for this purpose, particularly for rigid body equilibrium.^{2,3,10} This may be at least partially due to the fact that they are sufficient conditions for the maintenance, as opposed to the existence, of rigid body equilibrium. Thus, if the rigid body is initially in an equilibrium state, and if these two requirements are satisfied during a subsequent time-interval, then the rigid body will continue to remain in an equilibrium state during that interval.

$\bar{\mathbf{F}} = \bar{\mathbf{0}}$ & $\bar{\mathbf{M}}_G = \bar{\mathbf{0}}$
are necessary (not
sufficient) conditions
for the existence
of equilibrium.

Necessary & sufficient
conditions to guarantee
maintenance (not existence)
of mechanical system
equilibrium:

$$\begin{aligned}\bar{\mathbf{F}} &= \bar{\mathbf{0}} \\ \bar{\mathbf{M}}_G &= \bar{\mathbf{0}}\end{aligned}$$

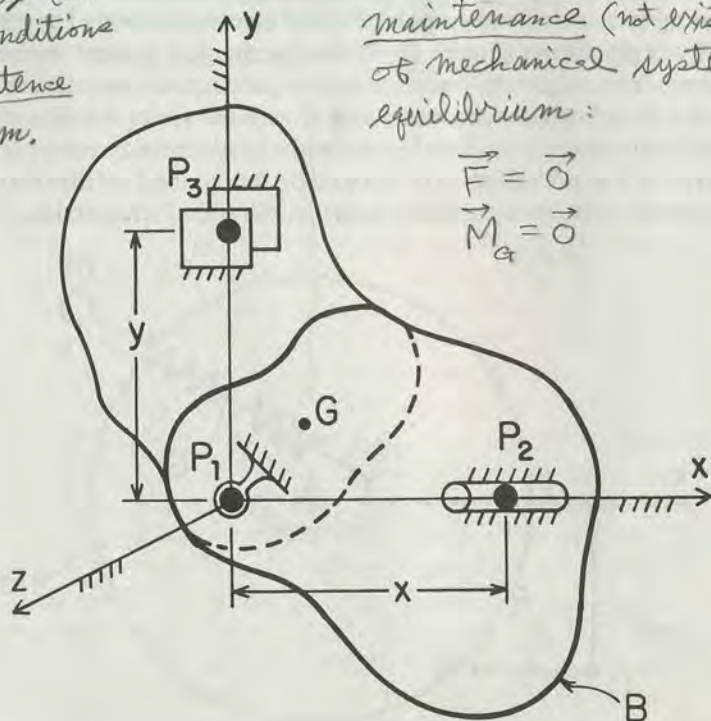


Fig. 3. Rigid body constrained at three non-collinear points P_1 , P_2 , and P_3 .

SUFFICIENCY PROOF

This result may be obtained using a proof by contradiction.^{6,7,8,9} To this end, consider a rigid body B in an equilibrium (rest) state in an inertial reference frame R and subject to an external force system with resultants $\bar{\mathbf{F}}$ and $\bar{\mathbf{M}}_G$ that satisfy $\bar{\mathbf{F}} = \bar{\mathbf{0}}$ and $\bar{\mathbf{M}}_G = \bar{\mathbf{0}}$. Assume that the persistence of these two kinetic conditions is not sufficient to maintain B in equilibrium. Additional forces are then applied to B at three noncollinear points P_i ($i = 1, 2, 3$) so as to constrain B from moving in R , or to maintain it in an equilibrium state. Referring to Figure 3, let a smooth ball-and-socket constraint force $\bar{\mathbf{F}}_1 = F_{1x}\bar{\mathbf{i}} + F_{1y}\bar{\mathbf{j}} + F_{1z}\bar{\mathbf{k}}$ be applied to B at P_1 , let a smooth pin constraint force $\bar{\mathbf{F}}_2 = F_{2y}\bar{\mathbf{j}} + F_{2z}\bar{\mathbf{k}}$ be applied to B at P_2 , where $\bar{\mathbf{r}}_{P_2/P_1} = x\bar{\mathbf{i}}$, and let a smooth two-sided constraint force $\bar{\mathbf{F}}_3 = F_{3z}\bar{\mathbf{k}}$ be applied to B at P_3 , where $\bar{\mathbf{r}}_{P_3/P_1} = y\bar{\mathbf{j}}$. Finally, let these three additional forces be equipollent to a resultant force $\bar{\mathbf{F}}'$ applied to B at G , together with a resultant moment $\bar{\mathbf{M}}'_G$ about G . (Two force systems are equipollent if their resultant forces are equal, and if their resultant moments about an arbitrary point are also equal.^{2,8,9}) The newly achieved equilibrium of B then necessarily implies that the new resultant force and moment about G must vanish, or that $\bar{\mathbf{F}} + \bar{\mathbf{F}}' = \bar{\mathbf{0}}$ and $\bar{\mathbf{M}}_G + \bar{\mathbf{M}}'_G = \bar{\mathbf{0}}$. However, since $\bar{\mathbf{F}} = \bar{\mathbf{0}}$ and $\bar{\mathbf{M}}_G = \bar{\mathbf{0}}$, these conditions separately imply that $\bar{\mathbf{F}}' = \bar{\mathbf{0}}$ and $\bar{\mathbf{M}}'_G = \bar{\mathbf{0}}$. These two vector equations, or their six independent scalar components, may be solved simultaneously to yield the result that the three additional constraint forces $\bar{\mathbf{F}}_i$ applied to B at points P_i all vanish! Thus, the original assumption, that $\bar{\mathbf{F}} = \bar{\mathbf{0}}$ and $\bar{\mathbf{M}}_G = \bar{\mathbf{0}}$ were not sufficient to maintain the rigid body in equilibrium, is contradicted, and must therefore be false. Consequently, $\bar{\mathbf{F}} = \bar{\mathbf{0}}$ and $\bar{\mathbf{M}}_G = \bar{\mathbf{0}}$ are sufficient to guarantee that rigid body equilibrium can be maintained. They are not sufficient, however, to guarantee that equilibrium exists, as the two examples previously cited demonstrate.

CONTRIBUTING FACTORS

Why do so many textbook writers continue to use the kinetic definition of equilibrium? Discounting the fact that commonly-accepted things tend to persist whether or not they are correct, the persistence of this misconception may be attributed to four main factors. The first is that the kinetic definition of equilibrium is, in fact, acceptable for single particle equilibrium. The second factor is that, if one simply ignores the subtle but significant change introduced when the requirements for the existence of rigid body equilibrium are replaced by the requirements for maintaining rigid body equilibrium, the kinetic definition of rigid body equilibrium also appears to be acceptable.

The third factor contributing to this problem seems to stem from the tendency by many writers to regard equilibrium as a force system property, rather than a kinematic system state. In particular, they treat equilibrium as if it were a special property of those external force systems that act on mechanical systems which themselves are in equilibrium. Although it is clearly possible to define something called "force system equilibrium" (e.g., a force system could be defined as being in equilibrium when its resultant force and its resultant moment about an arbitrary point both vanish), such a definition is ill-advised for at least two

reasons. First, force system equilibrium is an unnecessary concept. Force systems already have well-established and well-defined properties (i.e., their resultant forces and resultant moments about specific points), and a well-defined mechanical equivalence relation already exists to quantitatively compare force systems (i.e., equipollence). Thus, the basic definitions, properties, and theorems associated with equipollent force systems can be used to adequately describe the situation when the resultant force $\bar{F}$ and the resultant moment $\bar{M}_O$ about O simultaneously vanish (i.e., such force systems are equipollent to the null force system, or to the null wrench).

The second reason why "force system equilibrium" is an ill-advised concept is that it can easily lead to the mistaken and contradictory conclusion that mechanical systems having external force systems which are "in equilibrium" are themselves necessarily in a state of rest relative to the preferred inertial reference frame. This is an improper inference, as the two examples cited previously clearly demonstrate.

The fourth factor that seems to have contributed to the persistence of the kinetic definition of equilibrium is the decision by many textbook writers to omit much of the elegant logical structure that underlies the science of mechanics. This is indeed unfortunate because it not only denies students the opportunity to understand and appreciate the beauty of the carefully-reasoned arguments that have been used to extend Newton's laws for particle motion to the dynamical analysis of extremely complicated mechanical systems, but it also appears to have fostered a certain laziness and lack of rigor in defining fundamental mechanical quantities, such as the equilibrium state of a mechanical system.

CONCLUSION

Mechanical system equilibrium is a confused concept in the minds of many students, not only because it has been improperly defined in many textbooks, but also because it has a longstanding history of misuse. For example, the terms "static equilibrium" and "dynamic equilibrium" continue to be commonly-used terms in many mechanics texts, yet both exemplify abuse of the concept of equilibrium. The use of the qualifier "static" is clearly redundant, and is therefore unnecessary when analyzing the behavior of mechanical systems. The use of the adjective "dynamic" in the latter instance introduces an obvious paradox when juxtaposed with equilibrium because "dynamic" usually implies accelerated motion relative to an inertial reference frame.

The basis for most of the confusion concerning equilibrium is the prevalence of the kinetic definition of this system state in undergraduate statics textbooks. This definition is not only misleading, but substantially incorrect. Mechanical system equilibrium is properly defined as a kinematic system state, and it is not an attribute or property of a class of force systems. Force systems are properly assessed in terms of their resultant force and resultant moment vectors about specific points, and force systems may be quantitatively compared using the well-defined equipollence equivalence relation.

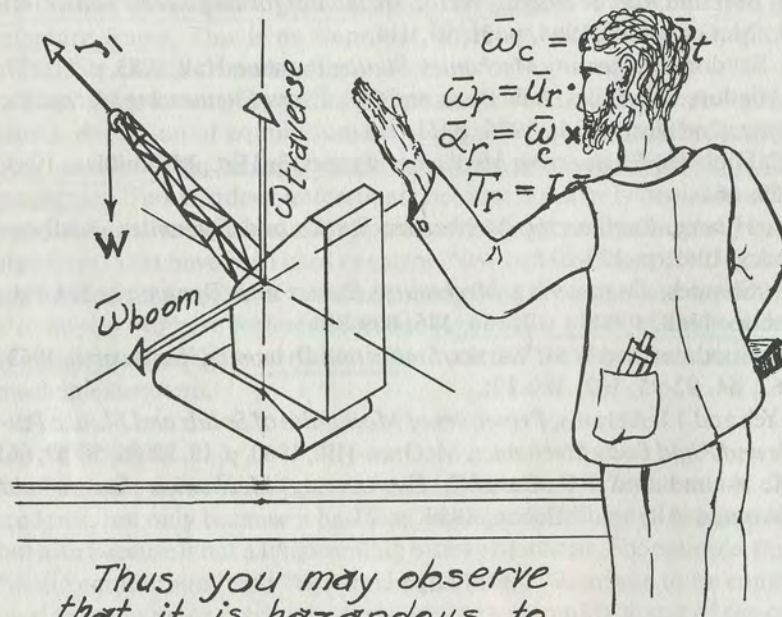
In order to minimize the adverse effects of the kinetic definition of equilibrium

where its presence cannot be easily eliminated, it would seem appropriate (1) to stress the proper kinematic definition of mechanical system equilibrium, (2) to present the logical arguments that lead to the discovery of necessary and sufficient kinetic conditions for the existence and/or maintenance of system equilibrium, and (3) to emphasize that engineers routinely use the necessary kinetic conditions for equilibrium to deduce useful information about the forces and stresses acting on mechanical systems that are known to be in equilibrium states.

REFERENCES

1. J.L. Meriam, *Engineering Mechanics: Statics*, SI Version, John Wiley & Sons, 1980, p. 38, 71, 82.
2. F.P. Beer and R.E. Johnston, *Vector Mechanics for Engineers: Statics*, 4th Ed., McGraw-Hill, 1984, p. 31, 97, 114.
3. B.I. Sandor, *Engineering Mechanics: Statics*, Prentice-Hall, 1983, p. 57, 137.
4. A. Higdon, W.B. Stiles, A.W. Davis and C.R. Evces, *Engineering Mechanics: Statics*, 2nd Vector Ed., 1976, p. 51, 113.
5. R.C. Hibbeler, *Engineering Mechanics: Statics*, 3rd Ed., Macmillan, 1983, p. 59, 162.
6. T.C. Huang, *Engineering Mechanics: Statics and Dynamics*, Addison-Wesley, 1968, p. 195-200.
7. I.H. Shames, *Engineering Mechanics: Statics and Dynamics*, 3rd Ed., Prentice-Hall, 1980, p. 19, 134, 146, 809-810.
8. L.E. Goodman and W.H. Warner, *Statics and Dynamics*, Wadsworth, 1963, p. 62, 64, 92-95, 102, 389-391.
9. H. Yeh and J.I. Abrams, *Principles of Mechanics of Solids and Fluids: Particle and Rigid Body Mechanics*, McGraw-Hill, 1960, p. 15, 52-54, 56-57, 66.
10. D.K. Anand and P.F. Cunniff, *Engineering Mechanics: Statics and Dynamics*, Allyn and Bacon, 1984, p. 73.

ALWAYS RELATE
MATHEMATICAL ANALYSIS
TO PRACTICAL CASES



Thus you may observe
that it is hazardous to
boom out while traversing a
heavily loaded crane!

Prof. Z. Freebody Snafu — Elmer L. Munger

A Challenge in Rigid-Body Dynamics

RAYMOND A. WILLEM
New Mexico State University

INTRODUCTION

The system being considered is shown in prototype form in Fig. 1 and diagrammatically in Fig. 2. It consists of two rigid bodies in contact with each other—a compound pendulum and the spring-biased bale of a common mouse trap. The mouse trap is fixed to a support surface and the pendulum is located so that the pendulum and bale can kinematically interact as the bale rotates through its arc of motion.

At the point of contact between the bale and pendulum there exists both normal and friction forces which constitute internal forces in the system, the friction force being significant to the performance of the system. The system is operated by releasing the spring-biased bale in contact with the pendulum and

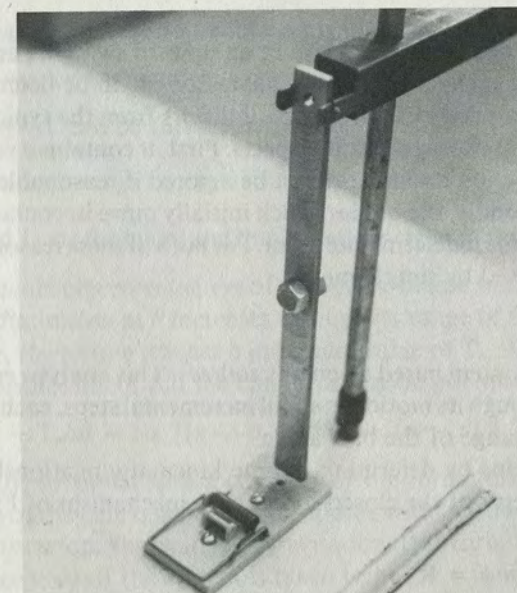


Fig. 1, System Prototype

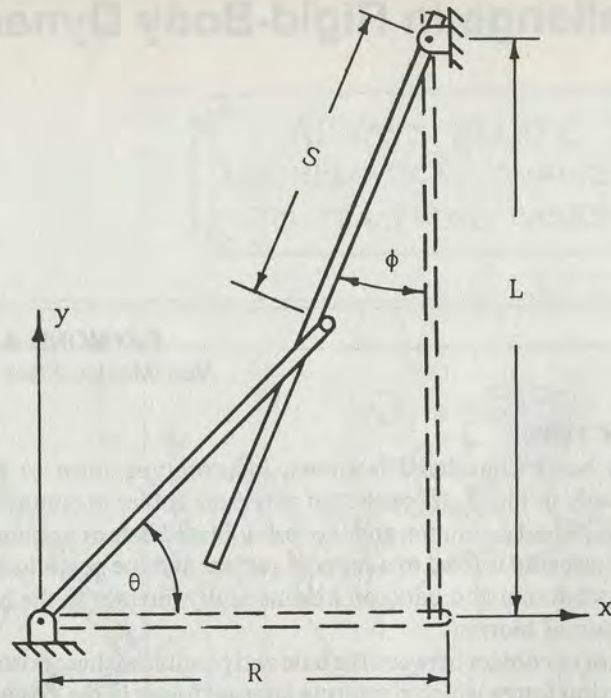


Fig. 2, System

permitting it to catapult the pendulum in an upward swinging arc. The total angle of that swing is the performance characteristic to be determined.

In spite of the simplicity of the system, it differs from the typical textbook dynamics problem in two significant respects. First, it contains a varying internal nonconservative force which can not be ignored if reasonable accuracy is to be achieved. Secondly, the bodies which initially move in contact with each other separate at some indeterminate point. For both of these reasons, the problem can not be solved by simple means.

ANALYSIS

An analysis of the system based on energy follows. This analysis considers the system to move through its motion in small incremental steps, each step involving the same $\Delta\theta$ change of the bale angle.

The analysis begins by determining some kinematic relationships for the system. The geometry of the closed loop in the mechanism of Fig. 2 can be expressed as

$$R\cos\theta + S\sin\phi = R \quad (1)$$

$$R\sin\theta + S\cos\phi = L \quad (2)$$

Combining equations 1 and 2 to eliminate S results in

$$\phi = \arctan\left(\frac{R - R\cos\theta}{L - R\sin\theta}\right) \quad (3)$$

Differentiating equation 3 with respect to time gives

$$\dot{\phi} = \dot{\theta} \left[\frac{LR\sin\theta - R^2(1 - \cos\theta)}{L^2 - 2LR\sin\theta + 2R^2(1 - \cos\theta)} \right] \quad (4)$$

Equation 3 indicates that the system has a single degree of freedom. If the system moves such that the value of θ changes by $\Delta\theta$, then, the corresponding change in ϕ ($\Delta\phi$) can be obtained using equation 3 in the form

$$\Delta\phi = \arctan\left[\frac{R - R\cos(\theta + \Delta\theta)}{L - R\sin(\theta + \Delta\theta)}\right] - \arctan\left[\frac{R - R\cos\theta}{L - R\sin\theta}\right] \quad (5)$$

Attention will now focus on the energy of the system. As the system moves through a $\Delta\theta$ displacement, the energy of the system changes according to the relationship

$$\Delta T + \Delta V_s + \Delta V_g = \Delta W_m \quad (6)$$

where

ΔT = kinetic energy change

ΔV_s = elastic potential energy change

ΔV_g = gravitational potential energy change

ΔW_m = work of nonconservative forces

The quantity ΔT can be further defined as

$$\Delta T = T_2 - T_1 \quad (7)$$

where T_1 and T_2 are the initial and final kinetic energies of the system respectively.

The torque-displacement curve of the spring-biased bale is shown in Fig. 3; the torque diminishes as θ increases through its range of 0 to π rad such that when $\theta = \pi$, the torque reaches a minimum value of T_o . As the system moves through a displacement $\Delta\theta$, the change in elastic potential energy is

$$\Delta V_s = -T_o\Delta\theta + \frac{1}{2}k[(\pi - \theta - \Delta\theta)^2 - (\pi - \theta)^2] \quad (8)$$

where k is the torsional spring constant of the spring-biased bale.

The mass of the bale is negligible compared with the mass of the compound pendulum, therefore, the change of gravitational potential energy produced by a $\Delta\theta$ displacement of the system is taken to be

$$\Delta V_g = mg\bar{X}_g[1 - \cos(\phi + \Delta\phi) - (1 - \cos\phi)] \quad (9)$$

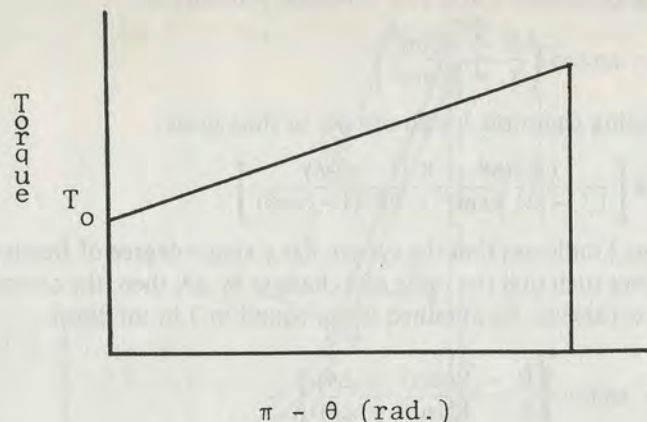


Fig. 3, Bale torque-displacement curve

where

m = compound pendulum mass

g = gravitational acceleration

$\bar{X}_g$ = distance from pivot to mass center of compound pendulum

The kinetic energy of the system at the beginning and end of the $\Delta\theta$ displacement are given by

$$T_1 = \frac{1}{2}I_p\dot{\phi}_1^2 + \frac{1}{2}I_b\dot{\theta}_1^2 \quad (10)$$

and

$$T_2 = \frac{1}{2}I_p\dot{\phi}_2^2 + \frac{1}{2}I_b\dot{\theta}_2^2 \quad (11)$$

respectively, where

I_p = mass moment of inertia of the pendulum

I_b = mass moment of inertia of the bale

$\phi_{1,2}$ = initial, final value

$\theta_{1,2}$ = initial, final value

At this point, equation 6 may be written in the form

$$T_2 = T_1 - \Delta V_s - \Delta V_g + \Delta W_m \quad (12)$$

Let it be assumed that the initial values of displacements and velocities ($\theta, \phi, \dot{\theta}_1, \dot{\phi}_1$) are known. Then ΔV_s in equation 12 can be determined with equation 8 and ΔV_g can be determined using equations 5 and 9. The kinetic energy term, T_1 , can be evaluated with equation 10 and a good approximation to ΔW_m can be

obtained by using the value of that quantity calculated for the previous $\Delta\theta$ increment of motion. Equation 12 can now be used to determine the unknown T_2 . Following that, $\dot{\theta}_2$ can be found with equations 4 and 11 and $\dot{\phi}_2$ can be determined using equation 4.

In preparation for considering the next $\Delta\theta$ increment of motion, a value of ΔW_m must be determined for the present increment. The non-conservative work represented by this term results from the friction force between the bale and pendulum shown in Fig. 4. The friction force, F , acting on the pendulum does no work because the mass particles of the pendulum at the contact point have no motion in the direction of the friction force. However, the friction force (reaction) acting on the bale does do work and it is given by the equation

$$W_m = - |F(S_2 - S_1)| \quad (13)$$

where

$$F = fN \quad (14)$$

and

$S_{1,2}$ = initial, final value

N = normal force between the bale and pendulum

f = kinetic coefficient of friction

To determine the normal contact force, N , the approximation is used that during the pendulum displacement increment, $\Delta\phi$, $\ddot{\phi}$ is constant. Then the value of $\ddot{\phi}$ can be found using the constant acceleration relationship

$$\dot{\phi}_2^2 = \dot{\phi}_1^2 + 2\Delta\phi(\ddot{\phi})$$

which can be written

$$\ddot{\phi} = (2\Delta\phi)^{-1} (\dot{\phi}_2^2 - \dot{\phi}_1^2) \quad (15)$$

The normal force is then determined considering the rate of change of angular momentum of the pendulum relative to its fixed pivot and equating it to the resultant moment. The equation expressing this is

$$N \left[\frac{S_1 + S_2}{2} \right] - mg\bar{X}_g \sin\left(\phi + \frac{\Delta\phi}{2}\right) = I_p\ddot{\phi}$$

which can be rearranged to

$$N = \frac{2}{S_1 + S_2} \left[I_p\ddot{\phi} + mg\bar{X}_g \sin\left(\phi + \frac{\Delta\phi}{2}\right) \right] \quad (16)$$

An expression for S_1 and S_2 can be obtained using equation 2.

$$S_1 = \frac{L - R\sin\theta}{\cos\phi} \quad (17)$$

$$S_2 = \frac{L - R\sin(\theta + \Delta\theta)}{\cos(\phi + \Delta\phi)} \quad (18)$$

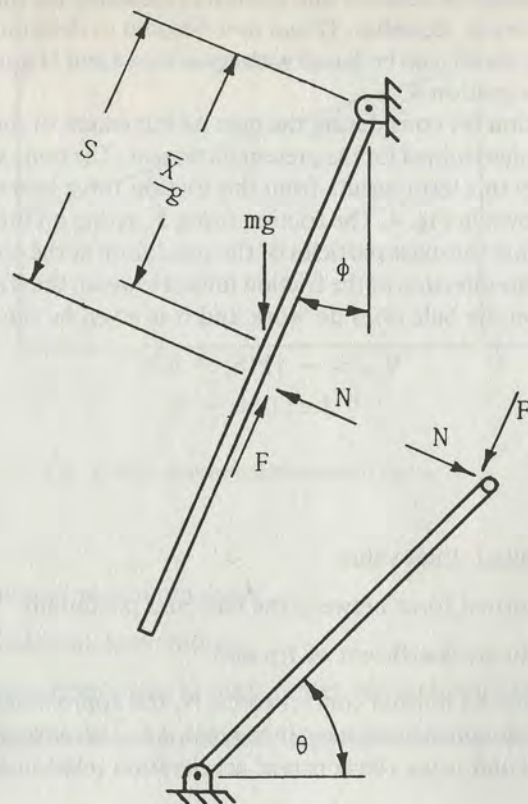


Fig. 4, Forces

Finally, the value of ΔW_m can be calculated using equations 13, 14, 17, and 18 and an assumed value for f .

The preceding analysis assumes the system has the configuration shown in Fig. 2 with the bale and pendulum in contact. The pendulum leaves the bale, however, at some indeterminate point and swings freely through the remainder of its motion. It is, therefore, necessary to determine the separation point and to use a different analysis after this point.

The separation point can be easily determined by finding the point at which the normal force between the pendulum and bale changes direction (sign). Beyond this point, the pendulum swings freely and will convert either part or all of its kinetic energy to gravitational potential energy.

Assuming the pendulum comes to rest during its upward swing, the maximum angle of pendulum rise, ϕ_f , can be found by equating the gain of potential energy to loss of kinetic energy. This can be expressed as

$$mg\bar{X}_g(\cos\phi_s - \cos\phi_f) = \frac{1}{2}I_p\dot{\phi}_s^2 \quad (19)$$

where

$$\phi_s = \phi \text{ at separation}$$

$$\dot{\phi}_s = \dot{\phi} \text{ at separation}$$

If equation 19 produces the result that $\cos\phi_f < -1$, then the assumption that the pendulum comes to rest is not met and it swings through the vertically upright position.

A computer program based on the preceding analysis has been developed and the results obtained are discussed in the next section.

An alternative formulation based on the rate of change of angular momentum alone has been developed but is not presented here. This formulation proved to be unstable early in the computer execution of the program. Prior to the instability, however, it gave results which agreed well with results obtained using the energy formulation.

RESULTS

A computer analysis of the prototype (Fig. 1) based on the energy formulation indicates that the motion of the system is significantly influenced by the friction force between the bale and the pendulum. This was verified by tests of the prototype in which the coefficient of friction between the two members was varied using different levels of lubrication and different surface roughnesses. Exact values of the friction coefficients achieved, however, were not known but could only be estimated using published values (1). Based on these estimated values, the results obtained by computer analysis appeared to be in good agreement with those obtained by test.

A summary of the computer results are given below. The measured and calculated system parameters are:

$$T_o = .73 \text{ lb. in.}$$

$$k = .668 \frac{\text{lb. in.}}{\text{radian}}$$

$$mg = .400 \text{ lb.}$$

$$I_p = .0230 \text{ lb. in. sec.}^2$$

$$I_b = .0228 \times 10^{-3} \text{ lb. in. sec.}^2$$

$$L = 8.55 \text{ in.}$$

$$R = 1.66 \text{ in.}$$

$$\bar{X}_g = 4.02 \text{ in.}$$

Values of the coefficient of friction and the corresponding values of maximum angle of pendulum swing are:

$$f \leq .2 \quad \text{swing through occurs}$$

$$f = .3 \quad \phi_f = 164.8^\circ$$

$$f = .4 \quad \phi_f = 147.9^\circ$$

$$f = .5 \quad (\text{instability occurred in the program})$$

CONCLUSION

In addition to providing an experience which will extend students understanding of dynamics and means of obtaining solutions to dynamics problems, exercises of the type described here are also real problems in predicting prototype performance—an important activity in engineering work.

REFERENCES

1. Standard Handbook for Mechanical Engineers, Seventh Edition, T. Baumeister, Editor, McGraw-Hill, New York, 1967.

Application of Computer Graphics to Engineering Mechanics Courses

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and

CLYDE E. WORK

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Mechanics professors and many others have traditionally relied on the chalkboard to reinforce the words they use, not only for presenting mathematical relationships but also to convey spacial relationships. Unfortunately, the two dimensional constraints of the chalkboard add to the limitations in class time and deficiencies in artistic talents of the teacher to prevent them from presenting their ideas as clearly as they would like.

A few use colored chalk, overhead projectors or models in their attempts to bring their presentations to life and to help their students visualize motion, stresses, deformation or flow that they may be analyzing. Most such aids remain two dimensional and static. Movies and video tapes which provide movement must be prerecorded and leave no opportunity for changing parameters on the spot in response to "what if?" questions.

Recent rapid advances in technology have opened new and exciting doors. Micro-computer hardware and powerful new software packages along with elegant projection techniques now make it possible to present, not only alphanumeric and symbolic characters, equations, and the like, but pictorial information as well.

The authors and their colleagues at Michigan Technological University have developed a system consisting of computer graphics hardware, software and projection equipment that are used to present a variety of two and three dimensional images in the classroom to enhance the student's ability to visualize and understand important concepts in Mechanics.

HARDWARE

The system used in classroom teaching consists of a Cromemco Z-2D Micro Computer with 128k of memory and an eleven megabyte hard disk. It handles

graphics through a Cromemco SDI interface and two 48k two port frame buffers driving an RGB monitor.

The hardware described is supported by a basic graphics software package, SDI Graphics Software, which is a product of Cromemco, Inc. This software provides the user of the Cromemco based system with a library of callable graphics subroutines in either the Basic or Fortran IV programming languages. To date our application software has been written in Fortran. This is primarily because the Cromemco Fortran is a compiler and thus program execution is much faster than with their basic interpreter and also because it is the language of choice in our engineering curriculum. Table I below gives a brief list of software used in developing the application software programs described in this paper.

The visuals thus created and controlled are presented to the class by means of an Aquastar IIIB large screen projector and a Lenco RGB to NTSC encoder. This equipment is capable of displaying sharp images up to 20 feet wide on a flat or curved screen on a front or rear projection basis.

Currently, the projector is set up in the rear projection mode for a classroom which seats 115 students. The room layout and arrangement of equipment is shown in Fig. 1. The keyboard and CRT monitor for the computer are on the teacher's table in the classroom. The room is partially darkened when the graphics is being displayed and a spotlight illuminates an area of the chalkboard to supplement the presentation on the screen. Since the room must also be used for classes that do not use the computer graphics, classes are moved into the room on days when the system is to be used. Although the equipment is all moderately portable, it is more convenient to move and focus people than the hardware.

DEVELOPING THE APPLICATION SOFTWARE

A TEAM APPROACH

Every effective application software program requires a considerable commitment of time. Faculty members already carrying heavy teaching loads have trouble finding time to develop such educational aids. In order to reduce the drain on faculty time, the authors have developed a team approach which optimizes the time and expertise requirements of such efforts.

Our development team(s) consist of a faculty member, a computer graphics

Table I
Graphics Development Software

Operating Systems	Cromemco CDOS, TM Cromix TM
Support Software	Cromemco SDI Graphics Software Cromemco FORTRAN IV Compiler Cromemco Z-80 Macro Assembler MTU Plotting Library MTU High Resolution Screen Dump

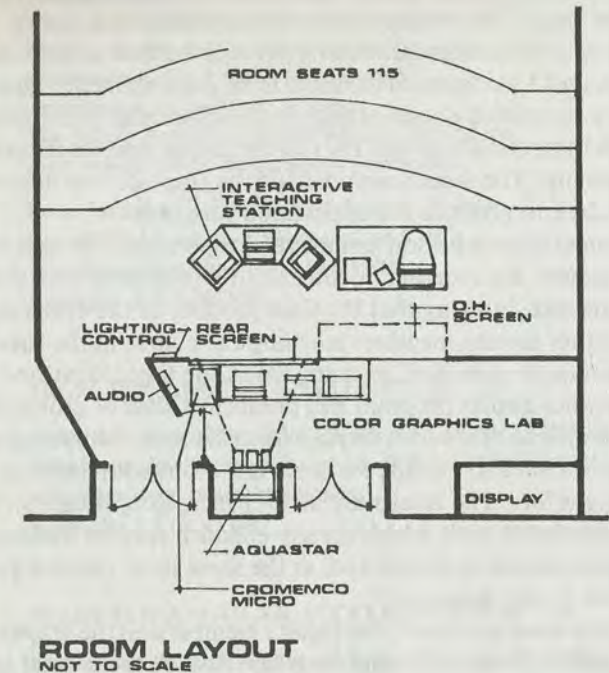


Figure 1

resource person, and a student programmer. The faculty member identifies a curriculum topic like those described later in this paper which he or she feels would be more effectively taught with the aid of interactive computer graphics illustration. The faculty member and the computer graphics resource person discuss it and define in detail how the display will look and how the program will be structured for the user interface. The graphics expert identifies the system hardware and software constraints that affect the desired display as well as structuring the "how" of inputting the variables that control the results displayed. Typical questions addressed are:

1. What should the basic display look like?
2. What information should be displayed and when should it appear in the display sequence?
3. Is color required, and if so, how do we apply it effectively?
4. Would animation aid in understanding the topic?
5. How much real time calculation must be done.
6. How should the user interact with the program to input information necessary for the desired results?
7. Do we need an iterative mode of program operation?

The result of this problem definition effort is a block structure of the program along with the documentation required to support its effective use and maintenance.

While these decisions are being finalized, a student is identified to join the development team. The student must: 1. be willing to work, 2. have some understanding of Fortran programming (have had at least an introductory Fortran course), and 3. understand the topic to be programmed. The student also needs to have completed a basic engineering graphics and descriptive geometry course. With these qualifications, the student can become an effective applications programmer. The faculty member and the graphics resource person work with the student to produce the application program.

The graphics resource person assumes the responsibility for the overall software development. He sees that documentation standards for software maintainability are met, insuring that the final product of the effort can and will be used by other faculty members teaching the course in the future.

It is important to note that, in many cases, the topic identified for the interactive graphics display program also points to a class or group of problems which are variations of the root display. In such cases, the basic graphics program can be extended to provide for these variations with minimal additional programming effort. The results are an ever increasing library of interactive graphics educational tools which greatly enhance student understanding of engineering mechanics concepts and, at the same time, increase productivity of the teacher in the classroom.

Through this team approach, the faculty member and the graphics resource person are able to focus their respective expertise on the critical areas of the program development effort with a minimal amount of time spent. The student contributes the bulk of the actual programming effort and, as a result, gains considerable expertise in applications programming and computer graphics. This experience also enhances the faculty member's understanding of computer graphics applications and capabilities. The result is a better informed faculty member with a clearer understanding of this new computer graphics technology. The authors feel the team approach provides a convenient mechanism for bringing faculty and students "up to speed" in computer graphics with a minimum of strain on all involved. The key is the graphics resource person who acts as a facilitator and interface in the development effort.

UTILIZATION IN MECHANICS

DYNAMICS

Wheel Rolling—The first application of the system was in the animation of a wheel rolling without slipping on a horizontal surface. A program was developed which shows the position of one point on the periphery of the wheel as the wheel rolls from left to right. As the highlighted point moves to each new position, each of its previous positions, the locus of which represents the trajectory of that point, is shown. At the beginning of the routine, the teacher makes the decision whether, once started, the wheel is to continue rolling along the plane or whether it is to move step by step on call. He chooses values of angular velocity and angular acceleration and designates which vectors are to be displayed. The text which appears on the screen of the monitor, the alter-

Wheel traverses the screen at a constant rate which is independent of input parameters. Display should be thought of as successive snapshots of the wheel with the vectors representing instantaneous values. The yellow dots that the circle leaves behind are spots where the point occupied previously.

Enter initial ANGULAR VELOCITY of wheel.
Use of REAL number between 0 and 10.
Type in number then press (ENTER).
Example: 5.3 (ENTER)

5.3

SELECTED ANGULAR VELOCITY IS 5.3

Enter the value of the ANGULAR ACCELERATION. Use a REAL number between 0 and 10. If 0 is selected the wheel will experience constant velocity.
Example: 2.6 (ENTER)

2.6

SELECTED ANGULAR ACCELERATION IS 2.6

Single Step through sequence? Enter (Y or N, ENTER)

Y

Show VELOCITY vector? Enter (Y or N ENTER)

Y

Show ACCELERATION vector? Enter (Y or N ENTER)

Y

Show INSTANT-CENTER LINE? Enter (Y or N ENTER)

Y

Figure 2.

natives available and choices made for a typical run are shown in Fig. 2. Typically, after showing the motion of the wheel with one point on the rim highlighted, a subsequent run displays the velocity vector for the point in each position.

The program develops the magnitude and direction of the velocity vector from an equation relating the velocity of the point to the velocity of the center of the wheel. This equation or algorithm is embedded in the graphics application software and "drives" the graphics callable subroutines which in turn drive the display hardware.

Upon command, the radius from the instantaneous center, the point where the wheel touches the surface on which it is rolling, is shown. The tangency of the velocity vector to the trajectory and its perpendicularity with respect to the line from the instantaneous center are dramatically pictured.

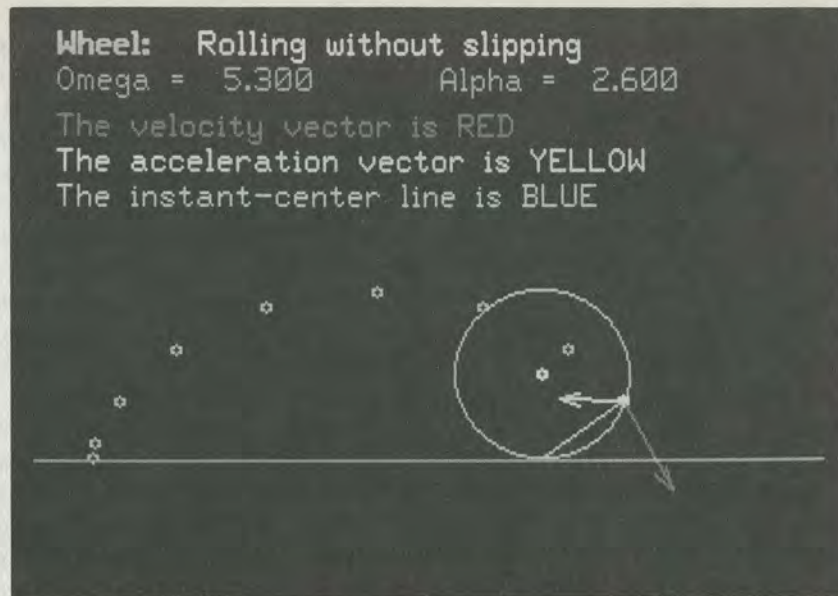


Figure 3.

One run is usually made with all vectors displayed simultaneously, each in a different contrasting color. Another run is made with the acceleration vector extinguished to emphasize the velocity vector. Alternatively, the velocity vector is extinguished and the acceleration vector displayed for each position. The non-zero value of acceleration at the point of contact and the influence of different values of ω and $\dot{\omega}$ come to life on the screen.

An alternative version of the program presents similar information for a spool type wheel rolling along a track on its inner radius with a rim extending past the track surface like that of a train wheel. The looping trajectory of a point on the periphery of the rim contrasts effectively with the trajectory of the point on the simple wheel. The vectors representing the velocity and acceleration provide the most informative contrasts for the points in the bottom position.

Spiral Path—To aid in the visualization and understanding of path coordinates, the motion of a point along a coplanar spiral path has also been animated. Programming the position vector in polar coordinates ($\theta = k_1 t$ and $r = k_2 t$), its time derivatives are expressed and displayed as components that are tangent and normal to the path.

The picture presents the continuous motion of the point as it travels out from the origin along the spiral path. Alternatively, the motion may be presented step by step in angular increments of 30° . As with the wheel, the locus of previous locations of the point is shown for each position as the point moves along its trajectory. A command to display the velocity shows the velocity vector with its changes in magnitude and direction as the motion progresses and

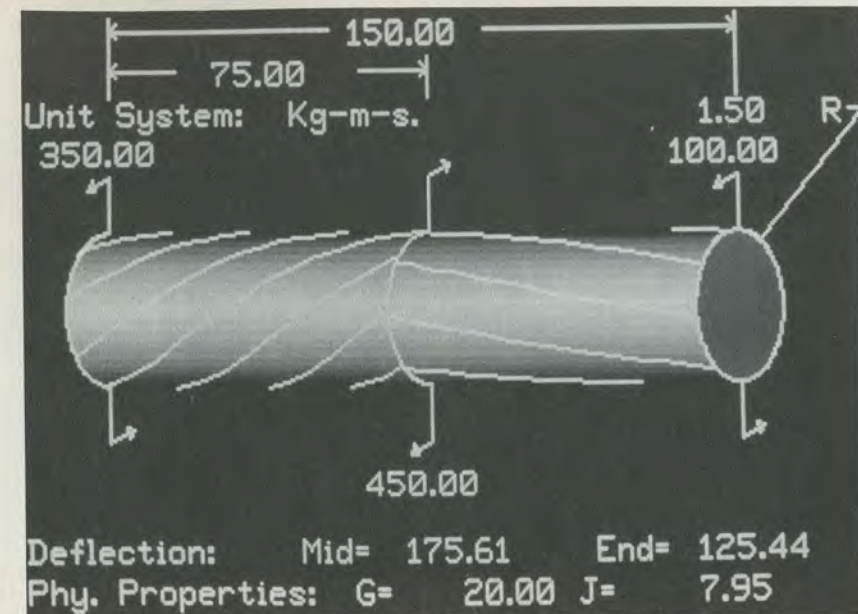


Figure 4. Torsion

demonstrates its tangency to the path.

A center of curvature command calls forth three normal lines, one at the location of the point at a given instant with other lines at points on either side which cross at the approximate location of the center of curvature. In motion, this routine shows the change of radius of curvature along the path.

The expanding of the tangential component of acceleration and shrinking of the normal component shows that the magnitude increases at a positive rate and that the rate of change of direction of the velocity vector decreases for the example chosen.

MECHANICS OF MATERIALS

Torsion—The twisting of a uniform cylindrical shaft is displayed in a colorful animation with a menu that solicits input of the torsional stiffness JG , the twisting moment applied at each of two planes and the relative spacing between them. The magnitude and sense of two couples, T_1 and T_2 are selected independently with the equilibrant couple T_3 adjusted automatically to match. The numerical values of all three as well as values of J , G and dimensions are displayed in inserts on the screen.

The animation shows the elements on the surface that are longitudinal lines when no moment is applied but which wrap around the surface in increasingly helical patterns as the moments build up. If $T_2 \neq 0$, the discontinuity at the section where it is applied is vividly pictured.

The overall length $\ell_1 + \ell_2$ is kept constant but the relative value of ℓ_1 and

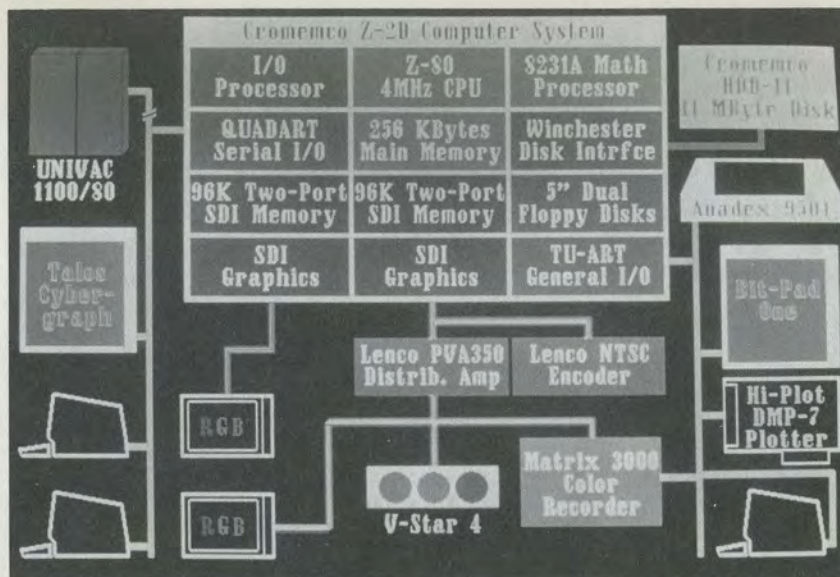


Figure 5. Microcomputer Based Graphics System Used to Produce the Programs and Graphics Described in this Paper.

ℓ_2 can be selected at will and the elastic stiffness JG for either the left or right portion can be adjusted by inputting different values.

Stress at a Point—A program is being developed which will display the stresses on a differential element in three dimensions and a plane inclined at an angle θ from one given plane (the x-plane). A two dimensional view of this state of stress is then pictures along with the corresponding Mohr's circle.

To aid the students in visualizing how the stresses on the inclined plane change as the angle that plane makes with the x-plane, the diagram is animated to show the differential element turning as the angle θ with the x-plane increases. For each position, the normal and shear stresses are shown to scale and the corresponding point on Mohr's circle is also shown with its coordinates highlighted.

STATICS

Moment of a Force—A program is being developed to help the student visualize the moment of a force in two and three dimensions, illustrating moments about lines through the point parallel to the coordinate axes.

CONCLUSIONS

Interactive computer graphics provides a new tool to enhance instruction in Engineering Mechanics courses. Hardware has been assembled at Michigan Tech to make this tool available in a classroom to project graphics displays to a class of a hundred. A team approach to develop software for specific examples has been used effectively. The classroom teacher (subject matter specialist) is

assisted by a graphics consultant (computer specialist and a student programmer) to optimize the efforts of each.

Examples have been developed for use in the teaching of undergraduate courses in dynamics, mechanics of materials and statics. The primary deterrents to rapid expansion of efforts to develop and use these ideas are the scheduling of the facilities in which they are presented to the class and the time of individuals to develop the software to display their ideas.

The Effect of Freshman Physics on Engineering Mechanics Preparation

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ABSTRACT

During the fall semester 1977 a mechanics readiness test was administered nationally to students entering their first course in engineering mechanics. The University of Idaho participated in that study and expanded upon its results by repeating the test at the completion of the course, and conducting a comparative analysis of the results. The results of that study have been reported separately.

At the time of the original study, our students had not taken an introductory engineering physics course prior to their enrollment in engineering mechanics. During the spring semester 1981 a freshman level non-calculus based physics course was introduced into the curriculum. This course will eventually become a required prerequisite for the mechanics offering; however, during the interim period students may take it prior to or concurrent with engineering mechanics. Approximately one half our freshmen students elected to take the course last spring.

At the beginning of the 1981 fall semester the original version of the National Mechanics Readiness Test was administered unannounced to every student entering our engineering mechanics course. Forty-eight percent of those students had taken and successfully passed the new physics course.

The test results of those students who took the physics course are statistically compared with the local and national ones of the previously administered test. The results show that the physics course did improve students' scores on the mechanics readiness test, over the non-physics students as well as over the 1977 national average. Those students who had not completed the physics course scored much like University of Idaho students in 1977, well below the national average.

BACKGROUND

During the fall semester 1977 a mechanics readiness test was administered nationally to students entering their first course in engineering. The University

of Idaho participated in this study and it was found that University of Idaho students' average score was 45 percent as compared to the national average of 51 percent. Our mechanics course is normally taken during the first semester of the student's sophomore year. The lack of an introductory physics course was cited as a possible cause for the below average results!

A freshman level non-calculus engineering physics course has since been introduced into the curriculum. This course will eventually become a required pre-requisite for engineering mechanics; however, during the interim it may be taken before or concurrently with engineering mechanics. Approximately one-half our freshman students elected to take the physics course during their first year. No other changes have been introduced into the freshman curriculum. The National Mechanics Readiness Test was administered, unannounced, on the first day of class to all students taking engineering mechanics during the fall 1981 semester. Since approximately one-half of this group had taken the freshman physics course we hoped to determine the benefits to University of Idaho engineering students of having a required physics pre-requisite, and to compare the level of preparation of the 1981 students with their counterparts from 1977.

DESCRIPTION OF THE TEST

The Mechanics Readiness Test was administered to 178 University of Idaho students enrolled in Engineering Science 211, Introduction to Mechanics, on the first day of class. The students had no previous knowledge of the test and were not allowed to use calculators. They were informed that their performance on the test would not affect their grade; the test was being used for course improvement and guidance purposes.

The students who took the test were first semester sophomores enrolled in an engineering curriculum. This curriculum consists of a common program during the freshman year (Table 1) that includes two semesters of calculus. The first semester of calculus, Math 180, is described in the July 1981 University of Idaho General Catalog as follows:

"180 Analytic Geometry and Calculus I. Functions, limits, continuity, differentiation, integration, appl. differentiation and integration of transcendental functions. Prereq: 2 yrs high school algebra (or 140) and 1 yr plane geometry and $\frac{1}{2}$ yr analytical trigonometry, or placement by test."

The second semester course is described as follows:

"Analytic Geometry and Calculus II. Differentiation and integration of transcendental functions, integration tech, general mean value theorem, numerical tech, and series. Prereq: 180."

Forty-eight percent of the students had taken Physics 210 which is described as follows:

"210 Engineering Physics. Kinematics and dynamics, work and energy, Newton's laws, oscillations, sound, geometric optics, physical optics, optical instruments."

The ES 211 Introduction to Mechanics course is the first basic engineering science course taken in the curriculum. It is a four semester credit course which

TABLE 1

Common Freshman Curriculum

First Semester			Cr
Course			
Chem	111	Prin of Chemistry	4
Eng	104	English Comp	3
Elective		Humanities-Social Sciences	3
Engr	101	Engr Graphics	2
Math	180	Anal. Geometry & Calculus I	4
PE	131	Freshman P.E.	1
			17
Second Semester			Cr
Course			
Chem	114	General Chemistry	4
Phys	210	Engr Physics I	3
Elective		Humanities-Social Sciences	3
Engr	131	Digital Comp. Prog.	2
Math	190	Anal. Geometry & Calculus II	4
PE	107	Freshman P.E.	1
			17

includes: resolution of forces; vector analysis; equilibrium; free body diagrams; centroids and moment of inertia; kinematics, kinetics, work energy and momentum methods for systems of particles. The format is three one-hour lectures per week accompanied by a two-hour problem laboratory each week. The text for the course is *Engineering Mechanics—Statics and Dynamics* by J.L. Meriam.

During the fall of 1981, six separate sections of this course were offered. Each section had an approximate enrollment of thirty students, and five different instructors were involved in the instruction of the six sections.

The students were given 45 minutes to complete the test, which consisted of twenty-nine multiple choice questions. Each question had five possible answers to choose from. Details on the construction of the test are presented in Snyder & Meriam. A copy of the test is appended to this paper.

The test covered several areas. The first four questions concerned the student's background and field of study. These questions enabled comparison of test results between certain groups of students. The core of the test included questions of both pre-college and college level mathematics which are generally regarded as prerequisites to the first mechanics course. The topics covered were trigonometry (questions 6, 7, 8), plane and analytic geometry (questions 5, 10, 11, 12, 13, 14, 15, 16, 17, 18, 21), scalar and vector algebra (22, 23, 24), and calculus (9, 25, 26, 27, 28, 29). The remaining questions concerned elementary physics and algebra.

The test was designed to be straightforward. Most of the questions are representative of the types of applications and operations encountered in basic mechanics. Therefore, the test was designed to reflect the geometrical and

physical applications of elementary mathematics, rather than the theoretical side of mathematics.

RESULTS

COMPARISON BASED ON COMPLETION OF PHYSICS COURSE

The average score on the test for students having completed Physics 210 was 54 percent or 13.5 correct responses on the 25 mathematics questions. The average score for those students who had not completed the physics course was 45 percent or 11.3 correct responses. The statistics are listed in Table 2.

TABLE 2

Comparison of Test Results Based on Completion of Physics Course

	Sample Size	Average Percent	Standard Deviation
Physics Completed	85	54	3.51
Physics Not Completed	93	45*	3.30

*Identical to the score obtained by 177 students in 1977 prior to the initiation of this physics course.

An analysis of variance of the two groups' results yields an F value of 1.13. Thus, it may be concluded that the populations are substantially the same. The "Z" value for comparing the means of the two groups is 11.43 which indicates that the students who had completed the physics course had a significantly better performance on the test than those students who had not completed physics.

COMPARISON BETWEEN 1977 and 1981 TEST RESULTS

The average score in 1977 was 45% with a standard deviation of 3.7.¹ The 177 students represented by these statistics are statistically identical to the 93 students who had not completed the physics course in 1977. Thus, the 85 students who had completed physics also represent a group better prepared for a mechanics course than their 1977 predecessors. These results indicate the desirability of requiring a prerequisite physics course to the introductory engineering course. These students were also compared to the 1977 national results. The 1977 national average was 51% with a standard deviation of 4.2.² This average was obtained from 9513 students. A means comparison results in a Z value of 5.3 indicating a significant difference. These Idaho students' performance was thus better than the 1977 national average.

Since the national average represents a composite score of students who had completed pre-requisite physics course as well as those who had not, the results of our comparison with this composite average are not surprising. This comparison is summarized in Table 3.

COMPARISON OF RESPONSES ON INDIVIDUAL QUESTIONS

Table 4 shows the individual responses on the 24 mathematics questions for the 1981 test group as well as the 1977 national and UI groups.

TABLE 3

Comparison of UI Results With 1977 National Results

Category	Sample Size	Average Percent	Standard Deviation
National 1977	9513	51*	4.2
UI 1977	177	45**	3.7
UI 1981 Freshman Physics	85	54*	3.5
UI 1981 Freshman Physics Not Completed	93	45**	3.3

*A comparison of these two groups shows the UI average score to be significantly higher.

**These two groups are statistically the same.

TABLE 4

Individual Question Results

Question	Correct Answer	1977 National	Freshman 1981 Physics Completed—UI	Freshman 1981 Physics Not Completed - UI	1977 UI
5	3	92.9	92.9	91.4	88.2
6	1	68.9	67.1	72.0	56.2
7	3	18.5	27.1	36.6	15.2
8	4	87.4	81.2	80.7	75.3
9	4	46.8	70.6	40.8	32.0
10	2	37.0	30.6	37.6	23.6
11	2	42.9	52.9	50.5	42.1
12	1	88.4	85.9	76.3	84.3
13	2	40.2	45.9	38.7	32.6
14	2	37.6	44.7	47.3	31.5
15	4	48.7	54.1	57.0	43.3
16	3	88.3	89.4	88.2	87.1
17	5	72.6	65.9	75.3	67.8
18	1	35.0	28.2	36.6	28.1
19	2	62.5	68.2	57.0	64.8
20	2	56.5	51.8	45.1	38.2
21	3	27.9	17.7	22.5	28.1
22	1	58.8	63.5	47.3	42.7
23	2	36.7	44.7	25.8	32.6
24	1	33.2	38.8	19.3	24.7
25	2	35.0	36.5	29.0	25.3
26	2	66.1	74.1	67.7	57.3
27	4	31.0	28.2	24.7	27.0
28	5	49.9	67.1	40.9	43.3
29	4	17.6	15.3	22.6	13.5

COMPARISON OF MATHEMATICAL CATEGORIES

Four categories of math problems were analyzed to compare the 1981 University of Idaho results for the effect of the physics course. The classification of the questions into the four categories was originally done by Snyder & Meriam

TABLE 5

Mathematics Category Results

Category	Question	1981 UI Average Physics Completed	1981 UI Average Physics Not Completed	1977 "Z" Value	1977 National Average	1988 UI Average
Trig.	6,7,8	58.4%	63.1%	1.2	57%	51%
Geometry	5,10,11 12, 13,14,15, 16,17,18, 21,22,23, 24	57.9%	56.5%	.4	55%	54%
Vectors	22,23,24	49.0%	30.8%	8.8	43%	37%
Calculus	9,25,26 27,29	44.9%	37.8%	2.3	40%	34%

and is discussed in their analysis of the national test. The results are summarized in Table 5, along with data from 1977.

Statistical means comparisons between the two groups of 1981 students indicate that significant improvement occurred in the areas of vectors and calculus for students who had completed the freshman physics course. The authors believe this results from their experience of having the abstract mathematical concepts integrated into actual physical applications and problems related to their area of interest. Means for the trigonometry and geometry problems were not significantly different for the two 1981 groups.

CONCLUSIONS

Based upon the results of this study, we conclude:

1. As a group, the University of Idaho 1981 sophomore students are no better prepared mathematically to undertake an engineering curriculum than our 1977 students.
2. Exposure of students to a physics course prior to their entry into engineering mechanics improves their mathematical preparation significantly.
3. Physical applications of vector and simple calculus concepts reinforce the abstract mathematics.

Although the introduction of a physics course into our freshman curriculum clearly improves the mathematical ability of our engineering students, their overall test results still remain well below our expectations. Opinions on an acceptable average score of 70—80% are consistent with those who composed the test. Even our improved results do not remotely approach this level of expectation. These results reinforce those of the national examination and substantiate the allegations that our students are seriously deficient in their understanding and ability to make use of even the elementary tools of mathematics.

CITED REFERENCES

1. Hager, Wayne, Steven L. Elgar, and Ping T. Sun, "An Analysis of the National Mechanics Readiness Test Based on Pre- and Post-Test Scores,"

presented at the American Society for Engineering Education Annual Conference, Baton Rouge, Louisiana, 1979.

2. Snyder, V.W. and J.L. Meriam, "The Mechanics Readiness Test—A Study of Student Preparedness," presented at American Society for Engineering Education Annual Conference, Vancouver, B. C., 1978.

MECHANICS READINESS TEST

Sponsored by the Mechanics Division of American Society for Engineering Education.

INSTRUCTIONS

- A. Your score on this test will be used to help you and your instructor evaluate your readiness to study mechanics.
- B. Notes, references, calculator, or slide rule may **not** be used.
- C. Time limit for the test—45 minutes.

- D. Print your name on the answer sheet. Fill in the appropriate space on the answer sheet provided for the *one correct alternative*. Make no marks on the test proper and return all exam pages and the IBM answer sheet to your instructor.

1. Your intended engineering major is:

- (1) Chemical Engineering
- (2) Civil Engineering
- (3) Electrical Engineering
- (4) Mechanical or Aeronautical Engineering
- (5) Other or not certain

2. Your last drawing or graphics course was in:

- (1) High School
- (2) College
- (3) None

3. If your school is on a semester basis, how many calculus courses have you completed?

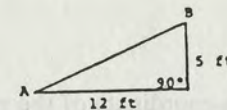
- (1) 0
- (2) 1
- (3) 2
- (4) 3
- (5) 4 or more

4. If your school is on a quarter basis, how many calculus courses have you completed:

- (1) 0
- (2) 1
- (3) 2
- (4) 3
- (5) 4 or more

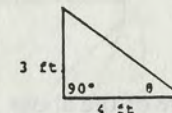
5. The length of AB is

- (1) 10 ft
- (2) $\sqrt{119}$ ft
- (3) 13 ft
- (4) 17 ft
- (5) None of the above



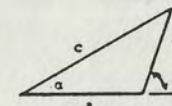
6. The value of $\sin \theta$ is

- (1) 0.6
- (2) 0.75
- (3) 0.8
- (4) 3
- (5) None of the above



7. The length of side c of the triangle shown is:

- (1) $\sqrt{a^2 + b^2 + 2ab \sin \alpha}$
- (2) $\sqrt{a^2 + b^2 - 2ab \cos \beta}$
- (3) $\sqrt{a^2 + b^2 + 2ab \cos \beta}$
- (4) $\sqrt{a^2 - b^2 - 2ab \cos \alpha}$



8. For an arbitrary angle θ

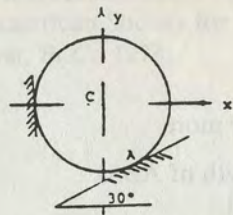
- (1) $\sin \theta + \cos \theta = 1$
- (2) $\sin \theta - \cos \theta = 1$
- (3) $\sin^2 \theta = 1 + \cos^2 \theta$
- (4) $\sin^2 \theta = 1 - \cos^2 \theta$
- (5) None of the above is true

9. As a close approximation, when an angle θ (expressed in radians) is very small, we may

- (1) replace $\cos \theta$ by zero
- (2) replace $\sin \theta$ by unity
- (3) replace $\cos \theta$ by θ
- (4) replace $\sin \theta$ by θ
- (5) None of the above

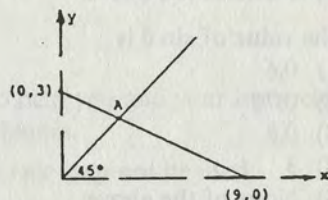
10. The 20-in diameter wheel with center at C rests against the two surfaces as shown. The perpendicular distance from point A to the y-axis is

(1) 2.5 in
 (2) 5 in
 (3) 5.77 in
 (4) 8.66 in
 (5) None of the above



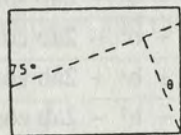
11. The x-coordinate of the point of intersection A of the two lines (which are not drawn to scale) is

(1) 2
 (2) 2.25
 (3) 3
 (4) 3.5
 (5) None of the above



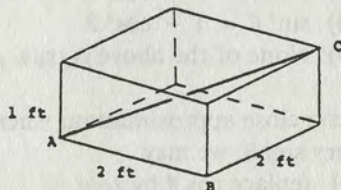
12. The two dotted lines in the rectangle are perpendicular to each other. Angle θ equals

(1) 15°
 (2) 20°
 (3) 25°
 (4) 30°
 (5) None of the above



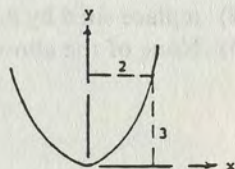
13. The rectangular box has a 2-ft by 2-ft-base and a height of 1 ft. The cosine of the angle between the diagonal AC and the edge AB is

(1) $2/5$
 (2) $2/3$
 (3) $1/\sqrt{2}$
 (4) $2/\sqrt{5}$
 (5) None of the above



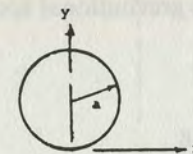
14. The equation of the parabola shown is

(1) $x/2 + y/3 = 2$
 (2) $y/3 = (x/2)^2$
 (3) $x/2 = (y/3)^2$
 (4) $y = 2x^2 - 5$
 (5) None of the above



15. The equation of the circle shown is

(1) $y^2 + (x - a)^2 = a^2$
 (2) $x^2 = y^2 + a^2$
 (3) $(x + y)^2 = a^2$
 (4) $x^2 + (x - a)^2 = a^2$
 (5) None of the above

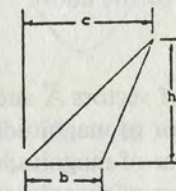


16. The area of a circle of radius r is

(1) $(\pi r)^2$
 (2) $\pi^2 r$
 (3) πr^2
 (4) $2\pi r$
 (5) None of the above

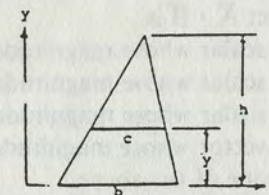
17. The area of the triangle is

(1) $\frac{b+c}{2} h$
 (2) $b(b/2)$
 (3) $h(b/3)$
 (4) hb
 (5) None of the above



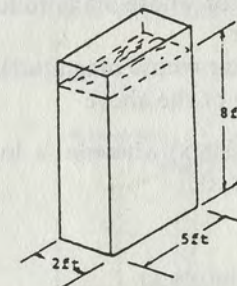
18. The horizontal distance c expressed in terms of y is

(1) $\frac{b}{h} (h - y)$
 (2) $\frac{h}{b} (h - y)$
 (3) $\frac{b}{h} y$
 (4) $\frac{h}{b} y$
 (5) None of the above



19. The tank shown is filled with salt water (specific weight 64 lb/ft^3) to a depth of 8 ft. The pressure exerted by the water on the bottom of the tank above atmospheric pressure is

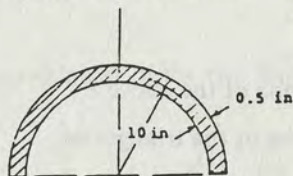
(1) 64 lb/ft^2
 (2) 512 lb/ft^2
 (3) 640 lb/ft^2
 (4) 14.7 lb/in^2
 (5) None of the above



20. The mass of a man on earth is 72 kg. If the man traveled to the moon where the gravitational acceleration is $1/6$ that on earth, his mass would be
- 12 kg
 - 72 kg
 - 432 kg
 - 72 times the moon's gravitational acceleration
 - None of the above

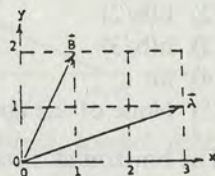
21. The shaded area of the half-circular ring is approximately

- 3.14 in²
- 8 in²
- 16 in²
- 32 in²
- None of the above



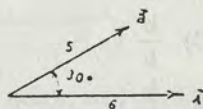
22. The sum of vectors $\vec{A}$ and $\vec{B}$ is

- a vector of magnitude 5
- a vector of magnitude $\sqrt{10}$
- a vector of magnitude $\sqrt{15}$
- a vector whose components are (3,2)
- None of the above



23. The magnitude of vectors $\vec{A}$ and $\vec{B}$ are 6 and 5, respectively. The dot product $\vec{A} \cdot \vec{B}$ is

- a scalar whose magnitude is 15
- a scalar whose magnitude is $15\sqrt{3}$
- a scalar whose magnitude is 30
- a vector whose magnitude is $15\sqrt{3}$
- None of the above



24. For the vectors in question 23, the cross product $\vec{A} \times \vec{B}$ is

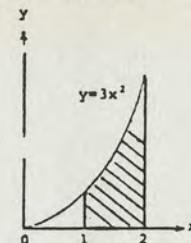
- a vector whose magnitude is 15 and whose direction is out from the paper
- a vector whose magnitude of $15\sqrt{3}$ and whose direction is out from the paper
- a vector whose magnitude is 15 and whose direction is into the paper
- a scalar whose magnitude is 15
- None of the above

25. If $y = \ln(\sin x)$ where $\ln = \log_e$, then dy/dx is

- $\ln(\cos x)$
- $\cot x$
- $\tan x$
- $\sin x \ln(\cos x)$
- None of the above

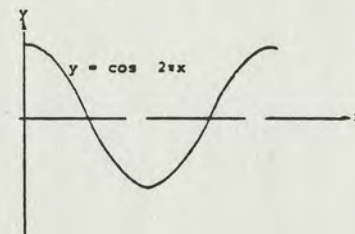
26. The shaded area under the curve $y = 3x^2$ between $x = 1$ and $x = 2$ is

- 1
- 7
- 8
- 9
- None of the above



27. The slope of the curve $y = \cos 2\pi x$ at $x = 1/4$ is

- 0
- 1
- 2π
- -2π
- None of the above



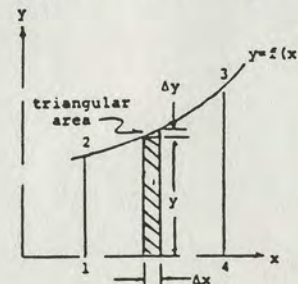
28. If the coefficients a , b , and c of the equation $ax^2 + bx + c = 0$ are positive constants, the smaller of the two roots is

- $-\frac{b}{a} + \frac{\sqrt{4ac - b^2}}{2a}$
- $-\frac{b}{2} - \frac{\sqrt{b^2 - 4ac}}{2}$
- $-\frac{b}{2a} - \frac{\sqrt{b^2 - 4ac}}{2b}$
- $-\frac{1}{2a} - \frac{\sqrt{b^2 - 4ac}}{2a}$

- (5) None of the above

29. To find the area 1-2-3-4 by integration, a vertical strip of area ΔA is chosen. In using only the cross-hatched area $y \Delta x$ for ΔA , the omission of the triangular area upon integration.

- introduces only a small but finite error when the curvature is small
- introduces a small but finite error regardless of the curvature
- introduces a small error only if $y = f(x)$ is of second or higher degree in x
- introduces no error
- None of the above is true.



NOTES